[7]Garner, H. C., Moore, A. W., and Wight, K. C., "The Theory of Interference Effects on Dynamic Measurements in Slotted-Wall Tunnels at Subsonic Speeds and Comparisons with Experiment," Aeronautical Research Council, Rept. and Memo 3500, 1966.

[8]Kinsler, L. E., Frey, A. R., Coppens, A. B., and Sanders, J. V., *Fundamentals of Acoustics*, Wiley, New York, 1982.

Enhancement of Frequency and Damping in Large Space Structures with Extendible Members

C. T. Sun* and R. T. Wang†
Purdue University, West Lafayette, Indiana 47907

Introduction

When disturbed, a large space structure (LSS) is likely to continue vibrating for some time because of its low frequencies and possibly small damping. Therefore, the objective of vibration control is to design the structure and its control to reduce dynamic responses in this structure. An effective way to achieve this goal is using damping augmentation that can be obtained by active or passive means. Juang[1] added a vibration absorber at the tip of a truss beam to produce the effect of passive damping augmentation. Balas[2] used an Euler-Bernoulli beam to illustrate active damping augmentation by direct velocity feedback of collocated/noncollocated sensors and actuators. Aubrun[3] used both direct feedback of velocity and displacement to change the modal frequencies and damping ratio of a structure. Meirovitch[4] extended the concept to modal space control of a distributed system.

Usually, in active damping-augmentation design, the sensors, actuators, and filters are mounted externally on the structure. The design requires a knowledge of elastic modal frequencies and modal shapes at the locations of sensors and actuators. However, these frequencies and shapes are very difficult to obtain in zero-gravity environment. Also, the vibrational modes of such structures are numerous, densely packed, and of low frequencies compared to the onboard controller bandwidth.

In contrast to the conventional control of a structure, the use of extendible truss members allows one to vary the control force internally. In this paper, the equations of motion for a truss with extendible members are derived. The effectiveness of using extendible truss members to tailor the vibrational characteristics of an LSS is investigated. Furthermore, a truss beam subjected to transient loading is used as an example to illustrate the effect of extendible truss members.

Formulation

Consider an extendible truss element consisting of an extending-contracting device which is assumed to be short in length compared with the length of the truss bar (see Fig. 1). The total elongation of the bar is given by

$$\Delta = \Delta_1 + \Delta_0 \tag{1}$$

Presented as Paper 88-2482 at the AIAA SDM Issues of the International Space Station Conference, Williamsburg, VA, April 21–22, 1988; received May 10, 1988; revision received Oct. 10, 1990; accepted for publication Nov. 29, 1990.

*Professor, School of Aeronautics and Astronautics. Associate Fellow AIAA.

†Graduate Student; currently Associate Professor, Department of Engineering Sciences, National Cheng Kung University, Taiwan, Republic of China.

where Δ_1 is the elongation of the elastic bar and Δ_0 is produced by the extending-contracting device. The corresponding total strain is

$$\epsilon = \epsilon_1 + \epsilon_0 \tag{2}$$

where $\epsilon_1 = \Delta_1/L$ and $\epsilon_0 = \Delta_0/L$. If damping is added to the element, then the stress in the truss bar is given by

$$\sigma(t) = E\epsilon(t) + c\dot{\epsilon}(t) - \sigma_0(t) \tag{3}$$

where $\sigma_0(t) = E\epsilon_0(t) + c\dot{\epsilon}_0(t)$ is the perturbing stress produced by the extension-contraction actuator, E is Young's modulus, $\dot{\epsilon}(t)$ is the strain rate, and c is a viscous damping coefficient.

The finite element formulation is achieved based on the following displacement function with respect to the local coordinates:

$$u(x, t) = u_1(t)(1 - x/L) + u_2(t)x/L$$

$$v(x, t) = v_1(t)(1 - x/L) + v_2(t)x/L \tag{4}$$

where u_i and v_i are the horizontal and vertical displacements, respectively, at the ith node.

Following the standard procedure, the equations of motion for the truss element are derived as

$$\begin{bmatrix} f_1 \\ q_1 \\ f_2 \\ q_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} A\sigma_0 = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} + \frac{cA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{v}_1 \\ \dot{u}_2 \\ \dot{v}_2 \end{bmatrix} + \frac{\rho AL}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{v}_1 \\ \ddot{u}_2 \\ \ddot{v}_2 \end{bmatrix} \tag{5}$$

where f_i and q_i are the vertical and horizontal nodal forces, respectively, at the ith joint. The axial force in the truss member is given by

$$S = \frac{EA}{L}[-1 \quad 0 \quad 1 \quad 0]\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} + \frac{cA}{L}[-1 \quad 0 \quad 1 \quad 0]\begin{bmatrix} \dot{u}_1 \\ \dot{v}_1 \\ \dot{u}_2 \\ \dot{v}_2 \end{bmatrix} - q_0 \tag{6}$$

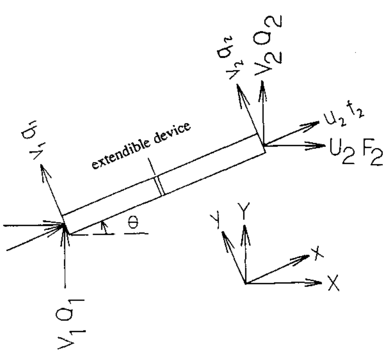


Fig. 1 A truss member with extension-contraction device.

where $q_0(=A\sigma_0)$ is the perturbing force in the truss member.

The system equations of motion for the entire truss referring to the global coordinate system can be obtained from Eq. (5) by the standard assemblage procedure. For convenience, we introduce the direction matrix $[B]$ as

$$[B] = [\{b_{12}\}\{b_{23}\}\cdots] \tag{7}$$

where $\{b_{ij}\}$ is an argumented direction vector for truss members between joint i and j obtained by adding zeros at the proper locations. Thus, the number of rows of $[B]$ is equal to the total nodal degrees of freedom, and the number of columns is equal to the number of truss members. Further details of the derivation are given in Ref. 5.

Using $[B]$, the assembled equations of motion can be expressed in the form

$$\{F\} + [B]\{q_0\} = [B][K][B]^T\{U\} + [B][D][B]^T\{\dot{U}\} + [M]\{\ddot{U}\} \tag{8}$$

where $\{F\}$ is the global nodal force vector, $\{U\}$ is the nodal displacement vector, $[M]$ is the system mass matrix, and $[K]$ and $[D]$ are diagonal matrices given by

$$[K] = \begin{bmatrix} (EA/L)_{12} & 0 & \cdots \\ 0 & (EA/L)_{23} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{9}$$

$$[D] = \begin{bmatrix} (cA/L)_{12} & 0 & \cdots \\ 0 & (cA/L)_{23} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{10}$$

in which quantities with subscript ij indicate properties of the truss member between node i and node j. In Eq. (8), the perturbing force vector $\{q_0\}$, which consists of the perturbing forces in the truss members, is defined as

$$\{q_0\} = \begin{Bmatrix} (q_0)_{12} \\ (q_0)_{23} \\ \vdots \end{Bmatrix} \tag{11}$$

The forces in all of the truss members can be expressed as

$$\{S\} = [K][B]^T\{U\} + [D][B]^T\{\dot{U}\} - \{q_0\} \tag{12}$$

Note that $[K]$ and $[D]$ are not to be confused with the usual stiffness and damping matrices that are given by

$$[K_0] = [B][K][B]^T, \qquad [D_0] = [B][D][B]^T \tag{13}$$

Alternately, the equations of motion can be written as

$$\{F\} + [B]\{q_0\} = [K_0]\{U\} + [D_0]\{\dot{U}\} + [M]\{\ddot{U}\} \tag{14}$$

Gain Effect on Frequency and Damping

The perturbing forces $\{q_0\}$ of truss members are to be controlled by varying the extensions $\{\Delta_0\}$ of the actuators as

$$\{q_0\} = [K]\{\Delta_0\} + [D]\{\dot{\Delta}_0\} \tag{15}$$

In this study, we consider the following relation

$$\{q_0\} = [G]\{S\} \tag{16}$$

where

$$[G] = \begin{bmatrix} g_{12} & 0 & \cdots \\ 0 & g_{23} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{17}$$

is the constant gain matrix. The relation given by Eq. (16) indicates that the perturbing force in each member is in proportion to the member force.

Substituting Eq. (27) into Eq. (22), we obtain

$$\{q_0\} = [G^*][K][B]^T\{U\} + [G^*][D][B]^T\{\dot{U}\} \tag{18}$$

and

$$\{S\} = [G^0][K][B]^T\{U\} + [G^0][D][B]^T\{\dot{U}\} \tag{19}$$

where

$$[G^*] = \begin{bmatrix} \dfrac{g_{12}}{1+g_{12}} & 0 & \cdots \\ 0 & \dfrac{g_{23}}{1+g_{23}} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$[G^0] = \begin{bmatrix} \dfrac{1}{1+g_{12}} & 0 & \cdots \\ 0 & \dfrac{1}{1+g_{23}} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \tag{20}$$

Comparing Eq. (15) with Eq. (18), we obtain

$$\{\Delta_0\} = [G^*][B]^T\{U\} \tag{21}$$

Eliminating $\{q_0\}$ from Eq. (14) using Eq. (18) yields

$$[M]\{\ddot{U}\} + [D_1]\{\dot{U}\} + [K_1]\{U\} = \{F\} \tag{22}$$

where the gain-augmented stiffness matrix $[K_1]$ and damping matrix $[D_1]$ are given by

$$[K_1] = [K_0] + [K_g], \qquad [D_1] = [D_0] + [D_g] \tag{23}$$

in which

$$[K_g] = -[B][G^*][K][B]^T, \qquad [D_g] = -[B][G^*][D][B]^T \tag{24}$$

Consider the special case where the gains of all truss members are

$$[G] = g[I] \tag{25}$$

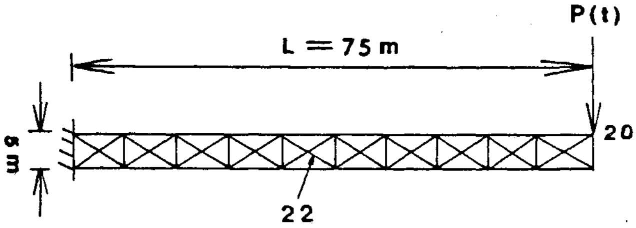
It follows that

$$[K_g] = -g(1+g)^{-1}[K_0], \qquad [D_g] = -g(1+g)^{-1}[D_0] \tag{26}$$

For free vibration, we obtain the natural frequency ω_i and the viscous damping factor μ_i for the ith mode as

$$\omega_i = \frac{1}{\sqrt{(1+g)}}\Omega_i, \qquad \mu_i = \frac{1}{\sqrt{(1+g)}}\mu_i^0 \tag{27}$$

where Ω_i is μ_i^0, the corresponding values at zero gain. From Eqs. (27), it is then concluded that both the natural frequencies and damping factors can be increased by using negative gain factors. Theoretically, if $g=-1$, then the structural system becomes rigid.



$E = 71.7 \times 10^9 \, N/m^2$, $\rho = 2768 \, kg/m^3$

$c = 4.0 \times 10^8 \, N_s/m^2$, $L_c = 7.5 \, m$

$L_g = 5.0 \, m$, $A_c = 90 \times 10^{-6} m^2$

$A_d = 40 \times 10^{-6} m^2$, $A_g = 60 \times 10^{-6} m^2$

Fig. 2 Geometry of the cantilevered truss (A_c, A_g, and A_d are the cross-sectional areas of the horizontal, vertical, and diagonal members, respectively).

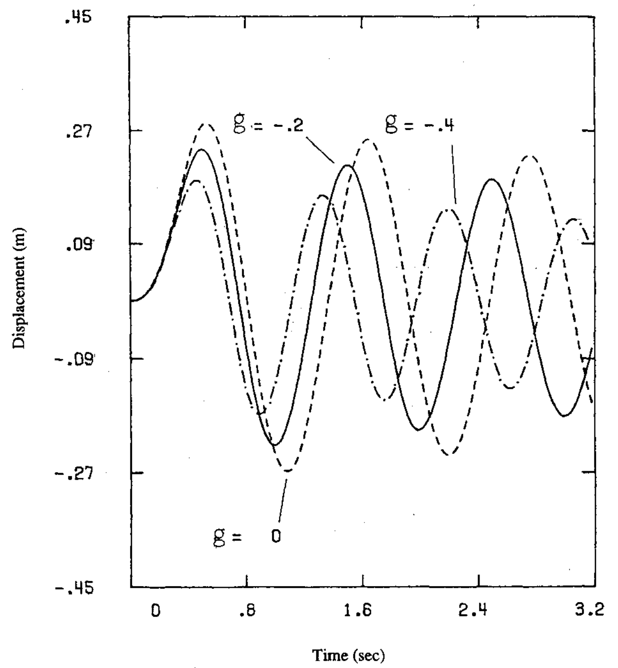


Fig. 3 Transient response history of the vertical displacement at the free tip of the damped structure.

Example

To study the transient forced response with feedback control, a concentrated force is assumed to act at the free tip as shown in Fig. 2. The forcing function is given as

$$P(t) = 100 \sin\left(\frac{\pi t}{T_0}\right) (N) \qquad t \le T_0$$

$$= 0 \qquad t > T_0$$

where $T_0 = 0.5$ s. The transient responses with respect to different gain factors are obtained by using the Newmark finite difference method.[6]

Figure 3 shows the time histories of the vertical displacement at the free tip of the truss with viscous damping. It is evident that with larger negative gains the frequency is increased and the amplitude decreased.

Conclusions

It has been demonstrated that effective structural stiffness and viscous damping can be augmented by the use of an extension-contraction actuator placed in each of the truss members. To achieve this purpose, the gain of the perturbing force of the actuator must be negative.

References

[1]Juang, J. N., "Optimal Design of a Passive Vibration Absorber for a Truss Beam," *Journal of Guidance, Control, and Dynamics,* Vol. 7, No. 6, 1984, pp. 733–739.

[2]Balas, M. J., "Active Control of Flexible Systems," *Journal of Optimization Theory and Applications,* Vol. 25, No. 3, 1978, pp. 415–436.

[3]Aubrun, J. N., "Theory of Control Structures by Low-Authority Controllers," *Journal of Guidance, Control, and Dynamics,* Vol. 3, No. 5, 1980, pp. 444–451.

[4]Meirovitch, L., and Baruch, H., "Control of Self-Adjoint Distributed-Parameter Systems," *Journal of Guidance, Control, and Dynamics,* Vol. 5, No. 1, 1982, pp. 60–66.

[5]Sun, C. T., and Wang, R. T., "Enhancement of Frequency and Damping in Large Space Structures with Extendible Members," *Proceedings of the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference,* AIAA, Washington, DC, 1988, pp. 1774–1780.

[6]Hughes, T. J. R., and Liu, W. K., "Implicit-Explicit Finite Elements in Transient Analysis: Stability Theory," *Journal of Applied Mechanics,* Vol. 45, No. 2, 1978, pp. 371-374.

Structural Damage Detection Using Modal Test Data

Tae W. Lim*
Lockheed Engineering and Sciences Company, Hampton, Virginia 23666

Introduction

FOR the assessment of structural integrity, the use of system identification techniques has been studied by many investigators.[1-3] Using the system identification techniques, the differences in structural dynamic characteristics between the undamaged and damaged structures are translated into the structural element stiffness changes, which reveal the structural damage. Assuming that the exact measured mode shapes of the damaged structure were available at every finite element degree of freedom (DOF) of the structure, Chen and Garba[1] and Smith and Hendricks[2] investigated the assessment of structural damage. Difficulties in identifying damaged members were reported. The stiffness matrix coefficients corresponding to the undamaged structural members were significantly affected, thereby making the damage detection uncertain. Hajela and Soeiro[3] proposed that static displacement information be incorporated to supplement the flexible modes for damage detection. For large space structures, such as the Space Station Freedom, static deflection may not be available because on-ground assembly testing of these structures is infeasible.

The influence of measurement inaccuracy on the damage detection was not addressed in any of the damage detection techniques described previously. In practice, the measured modes used for the damage detection are corrupted by the measurement noise and the modal parameter identification error. Also, the mode shapes are incomplete, i.e., available only at the measurement locations. This Note presents a systematic method that provides precise identification of the damage location and extent when the exact measured modes at every finite element DOF are used. Also, a procedure is presented to perform a damage detection with inaccurate, incom-

Received July 2, 1990; revision received Nov. 2, 1990; accepted for publication Nov. 9, 1990.

*Senior Engineer, Langley Program Office, 144 Research Drive. Member AIAA.